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Diagnosis and Prediction of Tipping Points in Financial Markets: Crashes and Rebounds

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Abstract

By combining (i) the economic theory of rational expectation bubbles, (ii) behavioral finance on imitation and herding of investors and traders and (iii) the mathematical and statistical physics of bifurcations and phase transitions, the log-periodic power law (LPPL) model has been developed as a flexible tool to detect bubbles. The LPPL model considers the faster-than-exponential (power law with finite-time singularity) increase in asset prices decorated by accelerating oscillations as the main diagnostic of bubbles. It embodies a positive feedback loop of higher return anticipations competing with negative feedback spirals of crash expectations. The power of the LPPL model is illustrated by two recent real-life predictions performed recently by our group: the peak of the Oil price bubble in early July 2008 and the burst of a bubble on the Shanghai stock market in early August 2009. We then present the concept of "negative bubbles", which are the mirror images of positive bubbles. We argue that similar positive feedbacks are at work to fuel these accelerated downward price spirals. We adapt the LPPL model to these negative bubbles and implement a pattern recognition method to predict the end times of the negative bubbles, which are characterized by rebounds (the mirror images of crashes associated with the standard positive bubbles). The out-of-sample tests quantified by error diagrams demonstrate the high significance of the prediction performance.

Keywords: financial bubble, crash, negative bubble, rebound, prediction, log-periodic power law, positive feedback, error diagram

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1. Introduction

Bubbles and crashes in financial markets are of global significance because of their effects on the lives and livelihoods of a majority of the world's population. In spite of this, the science to correctly identify bubbles in advance of their associated crashes produces fewer successful results than that used to treat baldness, choose 'quality' videos to watch or find a date on the internet. Instead, pundits and experts alike line up after the fact to claim that a particular bubble was obvious in hindsight. We present here a report on recent progress of the on-going effort of our group, the Financial Crisis Observatory at ETH Zurich (www.er.ethz.ch/fco/, to advance the science of understanding why, how and when bubbles form so that they can be identified before they spectacularly crash and spread misfortune to those who knowingly or unknowingly had bet on a long position.

The basis for our approach contradicts the accepted wisdom of the Efficient Market Hypothesis, which claims that large deviations from fundamental prices (i.e., bubbles and crashes) only exist when a new piece of information drops (exogenously) onto an unsuspecting market on a very short time scale. Instead, we claim that bubbles are the result of endogenous market dynamics over a much longer time scale—weeks, months and years. Because of the long build-up of these effects, bubbles can be identified by particular dynamical signatures predicted by our theoretical framework.

Among the well-documented history of financial bubbles and crashes over the past 400 years, through countless significantly different countries and kings, empires and economies, regulations and reform, there has been one consistent ingredient in all booms and busts: humans. Only human behavior has survived all attempts at preventing repeats in the wake of disastrous crashes. Much of the dynamics of the long time scales mentioned above is due to humans acting like humans: those without the knowledge imitate one another in the absence of a clearly better alternative and take refuge in the comfort of the crowd (herding) while those with the knowledge refute the masses and claim these noise traders are wrong.

We hypothesize that the signatures of this characteristic human behavior can be quantitatively identified. The imitation and herding behavior creates positive feedback in the valuation of an asset, resulting in a greater-than-exponential (power law) growth of the price time series. The tension and competition between the learned experts and the noise traders creates decorations on this power law growth comprising oscillations that are periodic in the logarithm of time. Log-periodic oscillations appear to our clocks as peaks and valleys with progressively smaller amplitudes and greater frequencies that eventually reach a point of no return, where the unsustainable growth has the highest probability of ending in a violent crash or gentle deflation of the bubble.

The log periodic power law (LPPL) model has thus been developed as a flexible tool to detect bubbles. This model combines (i) the economic theory of rational expectation bubbles, (ii) behavioral finance on imitation and herding of investors and traders and (iii) the mathematical and statistical physics of bifurcations and phase transitions. The LPPL model considers the faster-than-exponential (power law with finite-time singularity) increase in asset prices decorated by accelerating oscillations as the main diagnostic of bubbles. It embodies a positive feedback loop of higher return anticipations competing with negative feedback spirals of crash expectations.

The power of the LPPL model is illustrated by several recent real-life predictions performed by our group. We will present two examples in this paper: one is the peak of the Oil price bubble in early July 2008 and the other is the burst of a bubble on the Shanghai stock market in early August 2009. We then present the concept of "negative bubbles", which are the mirror

images of positive bubbles. We argue that similar positive feedbacks are at work to fuel these accelerated downward price spirals. We adapt the LPPL model to these negative bubbles and implement a pattern recognition method to predict the end times of the negative bubbles, which are characterized by rebounds (the mirror images of crashes associated with the standard positive bubbles). The out-of-sample tests quantified by error diagrams demonstrate the high significance of the prediction performance.

This paper is constructed as follows. We will give a general introduction to the LPPL model in section 2. In section 3, two successful real-life predictions mentioned above are discussed. Then we describe the phenomenon of negative bubbles and their termination in the form of rebounds. We develop a pattern recognition method based on the LPPL model and test its prediction performance in section 4. Section 5 concludes.

2. Log-periodic Power Law

2.1. Long time scale fermentation of bubbles

In sharp contrast to the Efficient Market Hypothesis (EMH) that crashes result from novel negative information incorporated in prices at short time scales, we build on the radically different hypothesis summarized by Sornette [1], that the underlying causes of the crash should be found in the preceding year(s). We define a bubble as a market regime in which the price accelerates "super-exponentially". The term "super-exponential" means that the growth rate of the price grows itself. A constant price growth rate (also called return) leads to an exponential growth, the normal average trajectory of most economic and financial time series. When the growth rate grows itself, the price accelerate hyperbolically. This growth of the growth rate is interpreted as being due to progressively increasing build-up of market cooperation between investors. As the bubble matures, its approaches a critical point, at which an instability can be triggered in the form of a crash, or more generally a change of regime. This critical point can also be called a phase transition, a bifurcation, a catastrophe or a tipping point. According to this "critical" point of view, the specific manner by which prices collapse is not the most important problem: a crash occurs because the market has entered an unstable phase and any small disturbance or process may reveal the existence of the instability. Think of a ruler held up vertically on your finger: this very unstable position will lead eventually to its collapse as a result of a small (or an absence of adequate) motion of your hand or due to any tiny whiff of air. The collapse is fundamentally due to the unstable position; the instantaneous cause of the collapse is secondary. In the same vein, the growth of the sensitivity and the growing instability of the market close to such a critical point might explain why attempts to unravel the proximal origin of the crash have been so diverse. Essentially, anything would work once the system is ripe.

What is the origin of the maturing instability? A follow-up hypothesis underlying this proposal is that, in some regimes, there are significant behavioral effects underlying price formation leading to the concept of "bubble risks". This idea is probably best exemplified in the context of financial bubble, where, fuelled by initially well-founded economic fundamentals, investors develop a self-fulfilling enthusiasm by an imitative process or crowd behavior that leads to the building of castles in the air, to paraphrase Malkiel [2].

Our previous research suggests that the ideal economic view, that stock markets are both efficient and unpredictable, may be not fully correct. We propose that, to understand stock markets, one needs to consider the impact of positive feedbacks via possible technical as well as behavioral mechanisms such as imitation and herding, leading to self-organized cooperation and the

development of possible endogenous instabilities. We thus propose to explore the consequences of the concept that most of the crashes have fundamentally an endogenous, or internal, origin and that exogenous, or external, shocks only serve as triggering factors. As a consequence, the origin of crashes is probably much more subtle than often thought, as it is constructed progressively by the market as a whole, as a self-organizing process. In this sense, the true cause of a crash could be termed a systemic instability.

2.2. Imitation and Herding Among Humans as the Cause of Bubbles

Humans are perhaps the most social mammals and they shape their environment to their personal and social needs. This statement is based on a growing body of research at the frontier between new disciplines called neuro-economics, evolutionary psychology, cognitive science and behavioral finance ([3], [4], [5]). This body of evidence emphasizes the very human nature of humans with its biases and limitations, opposed to the previously prevailing view of rational economic agents optimizing their decisions based on unlimited access to information and to computation resources.

We hypothesize that financial bubbles are footprints of perhaps the most robust trait of humans and the most visible imprint in our social affairs: imitation and herding (see Sornette [1] and references therein). Imitation has been documented in psychology and in neuro-sciences as one of the most evolved cognitive processes, requiring a developed cortex and sophisticated processing abilities. In short, we learn our basics and how to adapt mostly by imitation all through our life. It seems that imitation has evolved as an evolutionary advantageous trait, and may even have promoted the development of our anomalously large brain (compared with other mammals), according to the so-called "social brain hypothesis" advanced by R. Dunbar [6]. It is actually "rational" to imitate when lacking sufficient time, energy and information to make a decision based only on private information and processing, that is, most of the time. Imitation, in obvious or subtle forms, is a pervasive activity of humans. In the modern business, economic and financial worlds, the tendency for humans to imitate leads in its strongest form to herding and to crowd effects. Imitation is a prevalent form in marketing with the development of fashion and brands. Cooperative herding and imitation lead to positive feedbacks, that is, an action leads to consequences which themselves reinforce the action and so on, leading to virtuous or vicious circles.

The methodology that we have developed consists in using a series of mathematical and computational formulations of these ideas, which capture the hypotheses that (1) bubbles can be the result of positive feedbacks and (2) the dynamical signature of bubbles derives from the interplay between fundamental value investment and more technical analysis. The former can be embodied in nonlinear extensions of the standard financial Black-Scholes model of log-price variations [7, 8, 9, 10]. The later requires more significant extensions to account for the competition between (i) inertia separating analysis from decisions, (ii) positive momentum feedbacks and (iii) negative value investment feedbacks [8].

2.3. Positive Feedback Among Traders Leads to Power Law Growth in Asset Price

The idea of positive feedback has led us to propose that one of the hallmarks of a financial bubble is the faster-than-exponential growth of the price of the asset under consideration, as already mentioned. It is convenient to model this accelerated growth by a power law with a so-called finite-time singularity [11]. This feature is nicely illustrated by the price trajectory of the Hong Kong Hang Seng index from 1970 to 2000, as shown in Fig. 1. The Hong Kong financial

market is repeatedly rated as providing one of the most pro-economic, pro-entrepreneurship and free market-friendly environments in the world, and thus provides a textbook example of the behavior of weakly regulated liquid and striving financial markets. In Fig. 1, the logarithm of the price p(t) is plotted as a function of time (in linear scale), so that an upward trending straight line qualifies as exponential growth with a constant growth rate: the straight solid line corresponds indeed to an approximately constant growth rate of the Hang Seng index equal to 13.8% per year.

The most striking feature of Fig. 1 is not this average behavior but instead the obvious fact that the real market is never following and abiding to a constant growth rate. One can observe a succession of price run-ups characterized by growth rates ... growing themselves: this is reflected visually in Fig. 1 by transient regimes characterized by strong upward curvature of the price trajectory. Such an upward curvature in a linear-log plot is a first visual diagnostic of a faster than exponential growth (which of course needs to be confirmed by rigorous statistical testing). Such a price trajectory can be approximated by a characteristic transient finite-time singular power law of the form

$$\log p(t) = A + B(t_c - t)^m \tag{1}$$

where B < 0, 0 < m < 1 and t_c is the theoretical critical time corresponding to the end of the transient run-up (end of the bubble). Such transient faster-than-exponential growth of p(t) is our definition of a bubble. It has the major advantage of avoiding the conundrum of distinguishing between exponentially growing fundamental price and exponentially growing bubble price, which is a problem permeating most of the previous statistical tests developed to identify bubbles [12, 13]. The conditions B < 0 and 0 < m < 1 ensure the super-exponential acceleration of the price, together with the condition that the price remains finite even at t_c . Stronger singularities can appear for m < 0.

To see that faster-than-exponential growth is naturally related to positive feedback, let us consider the following simple presentation. Consider a population of animals of size x which grows with some constant rate k, i.e., dx/dt = k x. Then, growth is exponential in that x(t) = x $x(0)e^{kt}$. On the other hand, positive feedback in growth dynamics arises if the growth rate k itself depends on the population size x in that the growth rate k = k(x) increases with the population size. A particular simple example is the setting $k(x) \sim x^{m-1}$, where m > 1. Indeed, m-1can be regarded as a measure of the degree of cooperation within the population: the higher mis the larger is the degree of cooperation. In the case of no cooperation, growth dynamics is exponential. Positive multiplicative feedback generates growth which is faster than exponential. Indeed, due to growth dynamics given by $dx/dt = kx^m$, the size of the population growth exhibits a finite-time signlarity at t_c . This singularity is attained according to $x(t) = x(0)[1 - \tau]^{\frac{1}{1-m}}$, $0 < \tau < 1$ where $\tau = \frac{t}{\tau}$. Note that when m = 1, no cooperation, growth is indeed exponential and when m > 1, growth dynamics is in fact faster than exponential. It is remarkable that a critical time t_c emerges apparently out of nowhere. Actually, t_c is determined by the initial conditions and the structure of the growth equation. This emergence of t_c which depends on the initial conditions justifies its name in mathematical textbooks as a "movable singularity."

Many systems exhibit similar transient super-exponential growth regimes, which are described mathematically by power law growth with an ultimate finite-time singular behavior. An incomplete list of examples includes: planet formation in solar systems by runaway accretion of planetesimals, rupture and material failures, nucleation of earthquakes modeled with the slip-and-velocity, models of micro-organisms interacting through chemotaxis aggregating to form fruiting bodies, the Euler rotating disk, and so on. Such mathematical equations can actually

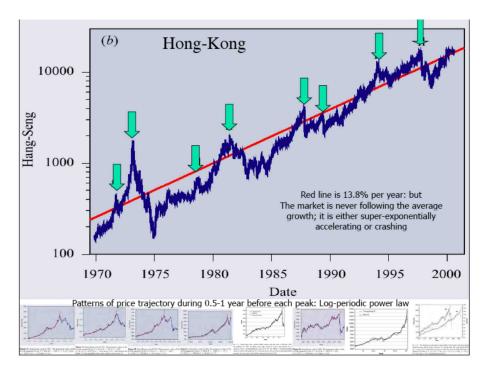


Figure 1: Trajectory of the Hong-Kong Hang Seng index from 1970 to 2000. The vertical log-scale together with the linear time scale allows one to qualify an exponential growth with constant growth rate as a straight line. This is indeed the long-term behavior of this market, as shown by the best linear fit represented by the solid straight line, corresponding to an average constant growth rate of 13.8% per year. The 8 arrows point to 8 local maxima that were followed by a drop of the index of more than 15% in less than three weeks (a possible definition of a crash). The 8 small panels at the bottom show the upward curvature of the log-price trajectory preceding each of these local maxima, which diagnose unsustainable bubble regimes, each of which culminates at its peak before crashing.

provide an accurate description of the transient dynamics not too close to the mathematical singularity where new mechanisms come into play. The singularity at t_c mainly signals a change of regime. In the present context, t_c is the end of the bubble and the beginning of a new market phase, possibly a crash or a different regime.

Such an approach may be thought at first sight to be inadequate or too naive to capture the intrinsic stochastic nature of financial prices, whose null hypothesis is the geometric random walk model ([2]). However, it is possible to generalize this simple deterministic model to incorporate nonlinear positive feedback on the stochastic Black-Scholes model, leading to the concept of stochastic finite-time singularities [7, 14, 15, 10, 16, 17]. Still much work needs to be done on this theoretical aspect.

Coming back to Fig. 1, one can also notice that each burst of super-exponential price growth is followed by a crash, here defined for the eight arrowed cases as a correction of more than 15% in less than 3 weeks. These examples suggest that the non-sustainable super-exponential price growths announced a "tipping point" followed by a price disruption, i.e., a crash. The Hong-Kong Hang Seng index shows that the average exponential growth of the index is punctuated by a succession of bubbles and crashes, which seem to be the norm rather than the exception.

2.4. Competition between different types of traders lead to log-periodic oscillations

More sophisticated models than Eq. (1) have been proposed to take into account the interplay between technical trading and herding (positive feedback) versus fundamental valuation investments (negative mean-reverting feedback). Accounting for the presence of inertia between information gathering and analysis on the one hand and investment implementation on the other hand [18], and taking additionally into account the coexistence of trend followers and value investing [8], the resulting price dynamics develop second-order oscillatory terms and boom-bust cycles. Value investing does not necessarily cause prices to track value. Trend following may cause short-term trend in prices but, together with value investing and inertia, also causes longer-term oscillations.

The simplest model generalizing (1) and including these ingredients is the so-called log-periodic power law (LPPL) model (see Sornette [1] and references therein). Formally, some of the corresponding formulas can be obtained by considering that the exponent m is a complex number with an imaginary part, where the imaginary part expresses the existence of a preferred scaling ratio γ describing how the continuous scale invariance of the power law (1) is partially broken into a discrete scale invariance [19]. The LPPL structure may also reflect the discrete hierarchical organization of networks of traders, from the individual to trading floors, to branches, to banks, to currency blocks. More generally, it may reveal the ubiquitous hierarchical organization of social networks recently reported [20] to be associated with the social brain hypothesis [6].

Examples of calibrations of financial bubbles with one implementation of the LPPL model are the 8 super-exponential regimes discussed above in Fig. 1: the 8 small insets at the bottom of the figure show the LPPL calibration on the Hang Seng index. Preliminary tests [1] suggest that the LPPL model provides a good starting point to detect bubbles and forecast their most probable end. Rational expectation models of bubbles a la Blanchard and Watson implementing the LPPL model [21, 22] have shown that the end of the bubble is not necessarily accompanied by a crash, but it is indeed the time where a crash is the most probable. But crashes can occur before (with smaller probability) or not at all. That is, a bubble can land smoothly, approximately one-third of the time, according to preliminary investigations [23]. Therefore, only probabilistic forecasts

can be developed. Probability forecasts are indeed valuable and commonly used in daily life, such as in weather forecast.

3. Successful case studies

3.1. The Oil Bubble of 2008

In [24], we have presented the prediction performed ex-ante and its post-mortem analysis of the bubble that has developed on oil prices in USD and in other major currencies. The exante diagnostic of the oil bubble was performed on the basis of the identification of unsustainable faster-than-exponential behavior during the course of 2007 until the mid-2008. We found support for the hypothesis that the oil price run-up in the first half of 2008 was amplified by speculative behavior of the type found during a bubble-like expansion. We also attempted to unravel the information hidden in the oil supply-demand data reported by two leading agencies, the US Energy Information Administration (EIA) and the International Energy Agency (IEA). We suggested that the found increasing discrepancy between the EIA and IEA figures provides a measure of the estimation errors. Rather than a clear transition to a supply restricted regime, we interpreted the discrepancy between the IEA and EIA as a signature of uncertainty. This is compatible with the idea that there is no better fuel than uncertainty to promote speculation. Our post-crash analysis confirmed that the oil peak in July 2008 occurred within the expected 80% confidence interval predicted ex-ante with data available in our pre-crash analysis.

The main result of our post-crash analysis is shown in Fig. 2. We calculated the 80% confidence interval for the critical time t_c of the end of the Oil bubble, which was found to be 17 May 2008 to 14 July 2008 and is shown as the shaded box in the main and inset plots of Fig. 2. The actual peak oil price was observed to be 3 July and the steep descent in price began on 11 July. Both dates are within the confidence interval calculated with the LPPL model using data through the last week of May. This confirms that the simple LPPL model was a successful predictor of the 2008 oil price bubble.

3.2. The Chinese Index Bubble of 2009

In the midst of the global financial crisis of the past year, when the prices of many assets and indexes fell, the Shanghai Composite Index defied financial gravity and continued climbing as it had done since October 2008.

On 10 July 2009 (using data through 9 July 2009), we submitted our prediction online to arXiv.org [25], in which we gave the 20%/80% (respectively 10%/90%) quantiles of the projected crash dates to be 17-27 July 2009 (respectively 10 July - 10 August 2009). This corresponds to a 60% (respectively 80%) probability that the end of the bubble occurs and that the change of regime starts in the interval within those time windows. Redoing the analysis 5 days later with data through 14 July 2009, the predictions tightened up with a 80% probability for the change of regime to start between 19 July and 3 August 2009 (unpublished). Our forecasts were featured in the press at, among other places, http://www.technologyreview.com/blog/arxiv/23839/.

On 29 July 2009, Chinese stocks suffered their steepest drop since November 2008, with an intraday bottom of more than 8% and an open-to-close loss of more than 5%. The market rebounded with a peak on 4 August 2009 before plummeting the following weeks. The Shanghai Index slumped 22 percent in August, the biggest decline among 89 benchmark indices tracked world wide by Bloomberg, in stark contrast with being the best performing index during the first

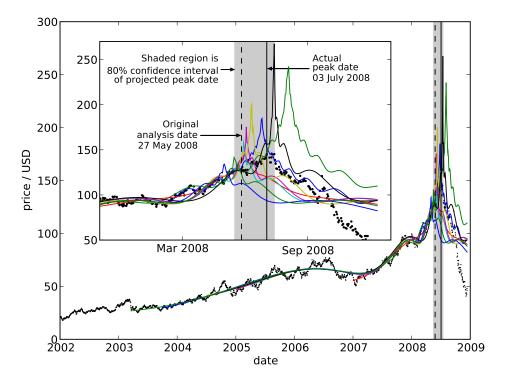


Figure 2: Time series of observed prices in USD of "NYMEX Light Sweet Crude, Contract 1" from the Energy Information Administration of the U.S. Government (see http://www.eia.doe.gov/emeu/international/Crude2.xls) and simple LPPL fits (see text for explanation). The oil price time series was scanned in multiple windows defined by (t_1, t_2) , where t_1 ranged from 1 April 2003 to 2 January 2008 in steps of approximately 3 months (see text) and t_2 was fixed to 23 May 2008. Shaded box shows the 80% confidence interval of fit parameter t_c for fits with t_c less than six months beyond t_2 . Also shown are dates of our original analysis in June 2008 and the actual observed peak oil price on 3 July 2008. Reproduced from [24].

half of 2009. We thus successfully predicted time windows for this crash in advance with the same methods used to successfully predict the peak in mid-2006 of the US housing bubble [26] and the peak in July 2008 of the global oil bubble [24]. The more recent bubble in the Chinese indexes was detected and its end or change of regime was predicted independently by two groups with similar results, showing that the model has been well-documented and can be replicated by industrial practitioners.

In [27], we presented a thorough post-crash analysis of this 2008-2009 Chinese bubble and, also, the previous 2005-2007 Chinese bubble in Ref. [28]. This publication also documents another original forecast of the 2005-2007 bubble (though there was not a publication on that, except a public announcement at a hedge-fund conference in Stockholm in October 2007). Also, it clearly lays out some of our many technical methods used in testing our analyses and forecasts of bubbles: the search and fitting method of the LPPL model itself, Lomb periodograms of residuals to further identify the log-periodic oscillation frequencies, (H, q)-derivatives [29, 30] and, most recently, unit root tests of the residuals to confirm the Ornstein-Uhlenbeck property of their stationarity [17]. Here, we reproduce the main figure documenting the advance prediction which included the peak of the bubble on 4 August 2009 in its 5-95% confidence limits. The curves are fitted by first order Laudau model:

$$\log p(t) = A + B(t_c - t)^m + C(t_c - t)^m \cos(\omega \ln(t_c - t) - \phi)$$
 (2)

4. Detection of Rebounds using Pattern Recognition Method

Until now, we have focused our attention on bubbles, their peaks and the crashes that often follow. We have argued that bubbles develop due to positive feedbacks pushing the price upward towards an instability at which the bubble ends and the price may crash.

But positive feedbacks do not need to be active just for price run-ups, i.e., for exuberant "bullish" market phases. Positive feedbacks may also be at work during "bearish" market regimes, when pessimism replaces optimism and investors sell in a downward spiral. Zhou and Sornette (2003) have developed a generalized Weierstrass LPPL formulation to model the 5 drops and their successive rebounds observed in the US S&P 500 index during the period from 2000 to 2002 [31].

Here, we go further and develop a methodology that combines a LPPL model of accelerating price drops, termed a "negative bubble," and a pattern recognition method in order to diagnose the critical time t_c of the rebound (the antithesis of the crash for a "positive bubble"). A negative bubble is modeled with the same tools as a positive bubble, that is, with the power law expression (1). But the essential difference is that the coefficient B is positive for a negative bubble (while it is negative for a normal positive bubble, as discussed above). The exponent m obeys the same condition 0 < m < 1 as for the positive bubbles. The positivity of B together with the condition 0 < m < 1 implies that the average log-price trajectory exhibits a downward curvature, expressing a faster-than exponential downward acceleration. In other words, the log-price trajectory associated with a negative bubble is the upside-down mirror image of the log-price trajectory of a positive bubble. Additional log-periodic oscillations are added to the LPPL model, which also account for the competition between value investors and trend followers.

We adapt the pattern recognition method of [32] to generate predictions of rebound times in financial markets. A similar method has been developed by Sornette and Zhou (2006) to combine

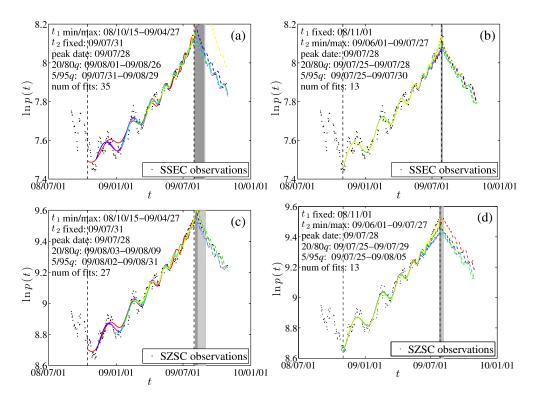


Figure 3: Daily trajectory of the logarithmic SSEC (a,b) and SZSC (c,d) index from Sep-01-2008 to Jul-31-2009 (dots) and fits to the LPPL formula (2). The dark and light shadow box indicate 20/80% and 5/95% quantile range of values of the crash dates for the fits, respectively. The two dashed lines correspond to the minimum date of t_1 and the fixed date of t_2 . (a) Examples of fitting to shrinking windows with varied t_1 and fixed t_2 = Jul-31-2009 for SSEC. The six fitting illustrations are corresponding to t_1 = Oct-15-2008, Nov-07-2008, Dec-05-2008, Jan-05-2008, Feb-06-2008, and Feb-20-2008. (b) Examples of fitting to expanding windows with fixed t_1 = Nov-01-2008 and varied t_2 for SSEC. The six fitting illustrations are associated with t_2 = Jun-01-2009, Jun-10-2009, Jun-19-2007, Jun-29-2007, Jul-13-2007, Jul-27-2007. (c) Examples of fitting to shrinking windows with varied t_1 and fixed t_2 = Jul-31-2009 for SZSC. The six fitting illustrations are corresponding to t_1 = Oct-15-2008, Nov-03-2008, Nov-26-2008, Dec-19-2008, Jan-14-2008, and Jan-23-2008. (d) Examples of fitting to expanding windows with fixed t_1 = Dec-01-2005 and varied t_2 for SZSC. The six fitting illustrations are associated with t_2 = Jun-01-2009, Jun-10-2009, Jun-19-2007, Jun-29-2007, Jul-13-2007, Jul-27-2007. Reproduced from Jiang et. al. [27].

the information provided by the LPPL model with a pattern recognition method for the prediction of the end of bubbles [33].

We analyze the S&P 500 index prices, obtained from Yahoo! finance for ticker 'GSPC' (adjusted close price)¹. The start time of our time series is 1950-01-05, which is very close to the first day of S&P 500 index (1950-01-03). The last day of our tested time series is 2009-06-03.

We first divide our S&P 500 index time series into different sub-windows (t_1, t_2) of length $dt \equiv t_2 - t_1$ according to the following rules:

- 1. The earliest start time of the windows is $t_{10} = 1950\text{-}01\text{-}03$. Other start times t_1 are calculated using a step size of $dt_1 = 50$ calendar days.
- 2. The latest end time of the windows is $t_{20} = 2009\text{-}06\text{-}03$. Other end times t_2 are calculated with a negative step size $dt_2 = -50$ calendar days.
- 3. The minimum window size is $dt_{min} = 110$ calendar days.
- 4. The maximum window size is $dt_{\text{max}} = 1500$ calendar days.

These rules lead to 11,662 windows in the S&P 500 time series.

Then we fit the log-price time series in each window with the LPPL model and get the corresponding parameters.

A rebound is defined by:

Rbd =
$$\{d \mid P_d = \min\{P_x\}, \forall x \in [d - 200, d + 200]\}$$
 (3)

where P_d is the adjusted closing price on day d. According to this definition, a rebound is said to have occurred on day d if condition (3) holds.

The prediction method uses a learning in-sample data set. Then, an out-of-sample dataset allows us to test the validity and success of the prediction method. We take all the fits before 1975-01-01 as learning set. In the learning set, if the critical time of a fit is near some of the rebounds, then we select this fit into Class I. In contrast, if there is not any rebound near the critical time of a fit, this fit will be classified into Class II. We then construct the statistics of all the parameters of the fits from Class I and Class II separately. We then compare the properties of these two sets of statistics for the two classes. The statistically significant differences between these properties allow us to define the features which are specific to each class.

Using these features associated with each class, we analyze every trading day before 1975-01-01. We collect all the fits with a critical time near this trading day, and find out the features of these fits. If most of the features in these fits belong to Class I, we interpret this as evidence that this trading day has a high probability to be a rebound. Otherwise, this trading day is not likely to be a rebound.

To quantify the probability that this trading day is a rebound, we develop the rebound alarm index as follows:

$$RI_{t} = \begin{cases} \frac{v_{t,I}}{v_{t,I} + v_{t,II}} & \text{if } v_{t,I} + v_{t,II} \ge 0\\ 0 & \text{if } v_{t,I} + v_{t,II} = 0 \end{cases}$$
 (4)

where $v_{t,I}$, $v_{t,II}$ represent the numbers of features of Class I and Class II respectively. From this definition, we can see that $RI_t \in [0, 1]$. If RI_t is high, then we can declare that this day has a high probability that the rebound will start.

¹http://finance.yahoo.com/q/hp?s=^GSPC

We use the features generated from the learning set, i.e. the fits before 1975-01-01, get the rebound alarm index before 1975-01-01 to check how the index performs. Fig. 4 shows the results. In this figure, the top panel shows the rebound alarm index for each trading day. The middle panel shows the times at which the real rebounds defined in (3) actually occurred. And the bottom panel is the log-price of the S&P 500 index. Fig. 4 shows that the rebound alarm index diagnosed the real rebounds quite well. A quantitation of what is meant by "quite well" will be given for the out-of-sample analysis performed below.

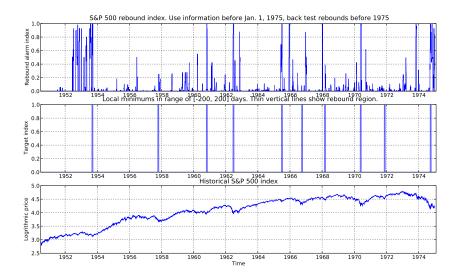


Figure 4: Top panel: rebound alarm index of the back tests on each day of our dataset before 1975-01-01. The rebound alarm index is defined in [0,1]. The higher the rebound alarm index, the more likely the rebound will happen. Middle panel: the actual rebounds in this period, as defined by (3). Lower panel: log-price of the S&P 500 index.

We now test how the rebound alarm index performs for ex-ante predictions. For this, we use the features learned from the learning set before 1975, then construct accordingly the rebound alarm index after 1975-01-01. The results are shown in Fig. 5. One can observe a satisfactory match between the periods when the rebound alarm index is large and the times when the rebounds actually occurred.

In order to quantify systematically and rigorously the quality of the performed predictions, we construct the associated error diagram as follows. We set a threshold T_h such that an alarm is declared when the rebound index passes above T_h . When this occurs, the alarm is set to last for 40 additional days. The total alarm set is thus the set of all days for which the rebound index is larger than T_h , augmented by the 40 consecutive days following the threshold days. A rebound is declared as having been predicted when it falls inside the total alarm set. For a given threshold T_h , we determine the fraction of days which fall in the alarm set (alarm period / total period). We also count the fraction of rebounds which have not been predicted. This defines the "failure to predict" equal to the fraction of missed rebounds. For a given threshold T_h , this gives one point in the plane where "failure to predict" is plotted as a function of the (alarm period / total period). Changing T_h from large to small values yields a line in the error diagram, which quantifies the

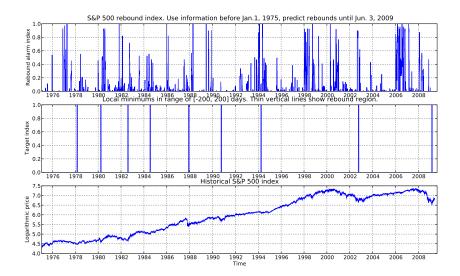


Figure 5: Same as figure 4 for the out-of-sample prediction period after 1975-01-01.

quality of the predictor in the way it addresses the trade-off between never missing a rebound target and declaring the smallest number of alarms (not "crying wolf" too much).

It is clear that, if an alarm is declared at all times, no rebound will be missed. This corresponds to the point (1,0). In contrast, declaring zero alarm will miss all targets. This corresponds to the point (0,1). In between, it is easy to see that the anti-diagonal line y=1-x corresponds to random predictions, such that the fraction of successfully predicted events is just the fraction of alarm time. In contrast, an ideal perfect predictor corresponds to the convergence to the origin (0,0), for which there is no failure to predict any rebound, while using an asymptotically vanishing fraction of time in alarms. A prediction system better than random falls below the anti-diagonal line y=1-x.

Fig. 6 presents the error diagrams obtained for different types of features. Fig. 7 also plots the error diagrams obtained in the back tests. The fast drop of failure to predict when the (alarm period / total period) is increased from zero is the signature of a very significant predictive ability.

5. Conclusion

We have presented partial results of an on-going ambitious project, that aims to test scientifically, rigorously and systematically the hypotheses that certain anomalous stock market regimes can be diagnosed in real-time and their tipping point can be determined better than chance. Specifically, we have presented two case studies of the recent bubble on the Oil price that culminated in early July 2008 and on the Shanghai stock market that crashes in August 2009. We have then shown how the proposed log-periodic power law (LPPL) model can be extended to so-called "negative" bubbles, these bearish market regimes where the price spirals down in an accelerated way, before rebounding. We have presented statistical tests in- and out-of-sample that demonstrate the strong predictive power of the proposed methodology.

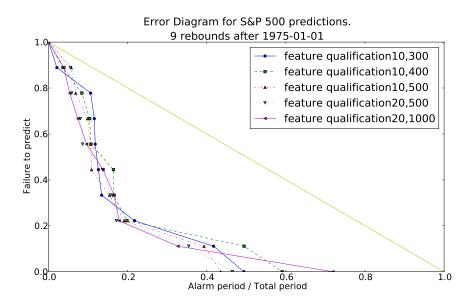


Figure 6: Error diagrams for predictions performed after Jan. 1, 1975 with different types of feature qualifications. Feature qualification α , β means that when the occurrence of a certain trait in Class I is more than α and less than β in Class II, then we call this trait a feature of Class I.

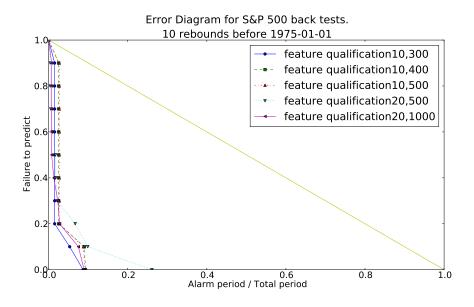


Figure 7: Same as figure 6 for the back tests performed before Jan. 1, 1975.

The research presented here is highly unconventional in financial economics and will swim against the convention that bubbles cannot be diagnosed in advance and crashes are somehow inherently impossible. But as Einstein once said: "Problems cannot be solved at the same level of awareness that created them." We thus propose a kind of Pascal's wager: is it really a big risk for the community to explore the possibility of changing the conventional wisdom and open new directions for the diagnostic of bubbles, ones that may eventually lead to important policy and regulatory implications?

This research may have indeed global impacts. We confront directly the wide-spread belief that crises are inherently unpredictable. If one can convince that some crises can be diagnosed in advance and, what is even more important, if one can quantify the associated uncertainties, this may help economists and policy makers develop new approaches to deal with financial and economic crises.

But piling up more and more case studies, we are convinced that the wealth of empirical results will foster many new theoretical insights for the understanding of financial bubbles. We envision that this will have a natural spill-over to the questions of how systemic risk and crisis depend on factors such as the increasing interdependencies in the global economic systems that arise from the development of new financial instruments. Central Banks, for example, could benefit from the models discussed here with a wide database of the national banks and of the firms and calculate the risks associated with these various economic agents. At a microeconomic level, banks could implement such models to supervise the activity of their branches to optimize risk management. The relatively small number of variables and parameters involved as well as the relative simplicity of the mathematics involved ensures the practical applicability of the model in any framework of network dependence and interactions. Banks could find it useful to monitor the exposition of correlation derivatives issued to immunize portfolios of liabilities. Financial institutions as well could appreciate the flexibility and real-time updating ability of our models to manage risk associated with large portfolios of assets.

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